

# Tau Physics

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## 1 Introduction

The  $\tau$  lepton is a member of the third generation which decays into particles belonging to the first and second ones. Thus,  $\tau$  physics could provide some clues to the puzzle of the recurring families of leptons and quarks. One naïvely expects the heavier fermions to be more sensitive to whatever dynamics is responsible for the fermion–mass generation. The pure leptonic or semileptonic character of  $\tau$  decays provides a clean laboratory to test the structure of the weak currents and the universality of their couplings to the gauge bosons. Moreover, the  $\tau$  is the only known lepton massive enough to decay into hadrons; its semileptonic decays are then an ideal tool for studying strong interaction effects in very clean conditions.

The last few years have witnessed a substantial change on our knowledge of the  $\tau$  properties [1, 2]. The large (and clean) data samples collected by the most recent experiments have improved considerably the statistical accuracy and, moreover, have brought a new level of systematic understanding.

## 2 Charged–Current Universality

The decays  $\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau$  and  $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$  are theoretically understood at the level of the electroweak radiative corrections [3]. Within the Standard Model (SM),

$$\Gamma(\tau^- \rightarrow \nu_\tau l^- \bar{\nu}_l) = \frac{G_F^2 m_\tau^5}{192\pi^3} f(m_l^2/m_\tau^2) r_{EW}, \quad (1)$$

where  $f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$ . The factor  $r_{EW} = 0.9960$  takes into account radiative corrections not included in the Fermi coupling constant  $G_F$ , and the non-local structure of the  $W$  propagator [3]. Using the value of  $G_F$  measured in  $\mu$  decay,  $G_F = (1.16637 \pm 0.00001) \times 10^{-5} \text{ GeV}^{-2}$  [3, 4], Eq. (1) provides a relation between the  $\tau$  lifetime and the leptonic branching ratios  $B_{\tau \rightarrow l} \equiv B(\tau^- \rightarrow \nu_\tau l^- \bar{\nu}_l)$ :

$$B_{\tau \rightarrow e} = \frac{B_{\tau \rightarrow \mu}}{0.972564 \pm 0.000010} = \frac{\tau_\tau}{(1.6321 \pm 0.0014) \times 10^{-12} \text{ s}}. \quad (2)$$

The errors reflect the present uncertainty of 0.3 MeV in the value of  $m_\tau$  [5, 6].

$m_\tau$	$(1777.05^{+0.29}_{-0.26})$ MeV
$\tau_\tau$	$(290.77 \pm 0.99)$ fs
$\text{Br}(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)$	$(17.791 \pm 0.054)\%$
$\text{Br}(\tau^- \rightarrow \nu_\tau \mu^- \bar{\nu}_\mu)$	$(17.333 \pm 0.054)\%$
$\text{Br}(\tau^- \rightarrow \nu_\tau \pi^-)$	$(11.02 \pm 0.09)\%$
$\text{Br}(\tau^- \rightarrow \nu_\tau K^-)$	$(0.690 \pm 0.025)\%$

Table 1: World average values for some basic  $\tau$  parameters.

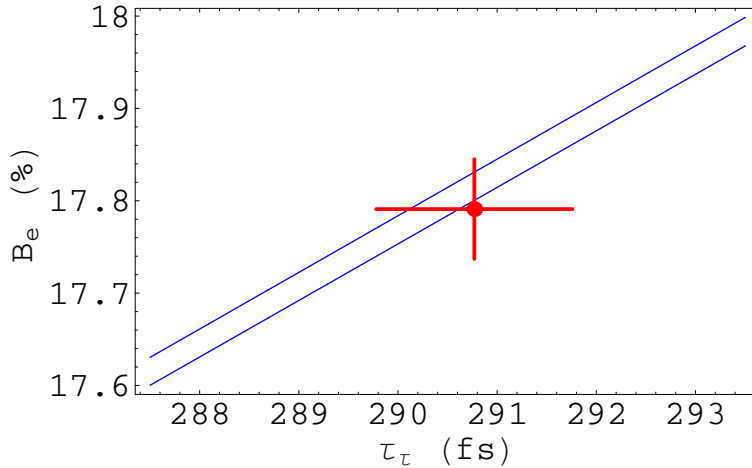


Figure 1: Relation between  $B_{\tau \rightarrow e}$  and  $\tau_\tau$ . The band corresponds to Eq. (2).

The relevant experimental measurements are given in Table 1. The predicted  $B_{\tau \rightarrow \mu}/B_{\tau \rightarrow e}$  ratio is in perfect agreement with the measured value  $B_{\tau \rightarrow \mu}/B_{\tau \rightarrow e} = 0.974 \pm 0.004$ . As shown in Fig. 1, the relation between  $B_{\tau \rightarrow e}$  and  $\tau_\tau$  is also well satisfied by the present data. The experimental precision (0.3%) is already approaching the level where a possible non-zero  $\nu_\tau$  mass could become relevant; the present bound [7]  $m_{\nu_\tau} < 18.2$  MeV (95% CL) only guarantees that such effect is below 0.08%.

These measurements can be used to test the universality of the  $W$  couplings to the leptonic charged currents. The ratio  $B_{\tau \rightarrow \mu}/B_{\tau \rightarrow e}$  constrains  $|g_\mu/g_e|$ , while  $B_{\tau \rightarrow e}/\tau_\tau$  and  $B_{\tau \rightarrow \mu}/\tau_\tau$  provide information on  $|g_\tau/g_\mu|$  and  $|g_\tau/g_e|$ . The present results are shown in Table 2, together with the values obtained from the ratios  $\Gamma(\pi^- \rightarrow e^- \bar{\nu}_e)/\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)$  [8] and  $\Gamma(\tau^- \rightarrow \nu_\tau P^-)/\Gamma(P^- \rightarrow \mu^- \bar{\nu}_\mu)$  [ $P = \pi, K$ ]. Also shown are the constraints obtained from the  $W^- \rightarrow l^- \bar{\nu}_l$  decay modes at the  $p\bar{p}$  colliders [6, 9] and LEP II [10]. The present data verify the universality of the leptonic charged-current couplings to the 0.15% ( $e/\mu$ ) and 0.23% ( $\tau/\mu, \tau/e$ ) level.

	$ g_\mu/g_e $	$ g_\tau/g_\mu $	$ g_\tau/g_e $
$B_{\tau\rightarrow\mu}/B_{\tau\rightarrow e}$	$1.0009 \pm 0.0022$	—	—
$B_{\tau\rightarrow e} \tau_\mu/\tau_\tau$	—	$0.9993 \pm 0.0023$	—
$B_{\tau\rightarrow\mu} \tau_\mu/\tau_\tau$	—	—	$1.0002 \pm 0.0023$
$B_{\pi\rightarrow e}/B_{\pi\rightarrow\mu}$	$1.0017 \pm 0.0015$	—	—
$\Gamma_{\tau\rightarrow\pi}/\Gamma_{\pi\rightarrow\mu}$	—	$1.005 \pm 0.005$	—
$\Gamma_{\tau\rightarrow K}/\Gamma_{K\rightarrow\mu}$	—	$0.981 \pm 0.018$	—
$B_{W\rightarrow l}/B_{W\rightarrow l'} \quad (p\bar{p})$	$0.98 \pm 0.03$	—	$0.987 \pm 0.025$
$B_{W\rightarrow l}/B_{W\rightarrow l'} \quad (\text{LEP2})$	$1.002 \pm 0.016$	$1.008 \pm 0.019$	$1.010 \pm 0.019$

Table 2: Present constraints on charged-current lepton universality.

### 3 Neutral-Current Universality

In the SM, all leptons with equal electric charge have identical couplings to the  $Z$  boson. This has been tested at LEP and SLC [10], by measuring the total  $e^+e^- \rightarrow Z \rightarrow l^+l^-$  cross-section, the forward-backward asymmetry, the (final) polarization asymmetry, the forward-backward (final) polarization asymmetry, and (at SLC) the left-right asymmetry between the cross-sections for initial left- and right-handed electrons and the left-right forward-backward asymmetry.  $\Gamma_l \equiv \Gamma(Z \rightarrow l^+l^-)$  determines the sum  $(v_l^2 + a_l^2)$ , where  $v_l$  and  $a_l$  are the effective vector and axial-vector  $Z$  couplings, while the ratio  $v_l/a_l$  is derived from the asymmetries which measure the average longitudinal polarization of the lepton  $l^-$ :

$$\mathcal{P}_l \equiv \frac{-2v_l a_l}{v_l^2 + a_l^2}. \quad (3)$$

The measurement of the final polarization asymmetries can (only) be done for  $l = \tau$ , because the spin polarization of the  $\tau$ 's is reflected in the distorted distribution of their decay products. Thus,  $\mathcal{P}_\tau$  and  $\mathcal{P}_e$  can be determined from a measurement of the spectrum of the final charged particles in the decay of one  $\tau$ , or by studying the correlated distributions between the final decay products of both  $\tau$ 's [11].

Tables 3 and 4 show the present experimental results. The data are in excellent agreement with the SM predictions and confirm the universality of the leptonic neutral couplings. The average of the two  $\tau$  polarization measurements,  $\mathcal{A}_{\text{Pol}}^{0,\tau}$  and  $\frac{4}{3}\mathcal{A}_{\text{FB,Pol}}^{0,\tau}$ , results in  $\mathcal{P}_l = -0.1450 \pm 0.0033$  which deviates by  $1.5\sigma$  from the  $\mathcal{A}_{LR}^0$  measurement. Assuming lepton universality, the combined result from all leptonic asymmetries gives

$$\mathcal{P}_l = -0.1497 \pm 0.0016. \quad (4)$$

Figure 2 shows the 68% probability contours in the  $a_l$ - $v_l$  plane, obtained from a combined analysis [10] of all leptonic observables. Lepton universality is now tested

	$e$	$\mu$	$\tau$	$l$
$\Gamma_l$ (MeV)	$83.90 \pm 0.12$	$83.96 \pm 0.18$	$84.05 \pm 0.22$	$83.96 \pm 0.09$
$\mathcal{A}_{\text{FB}}^{0,l}$ (%)	$1.45 \pm 0.24$	$1.67 \pm 0.13$	$1.88 \pm 0.17$	$1.701 \pm 0.095$

Table 3: Measured values [10, 12] of  $\Gamma_l$  and  $\mathcal{A}_{\text{FB}}^{0,l}$ . The last column shows the combined result (for a massless lepton) assuming lepton universality.

$-\mathcal{A}_{\text{Pol}}^{0,\tau} = -\mathcal{P}_\tau$	$0.1425 \pm 0.0044$	$\frac{4}{3}\mathcal{A}_{\text{FB,LR}}^{0,e} \Rightarrow -\mathcal{P}_e$	$0.1558 \pm 0.0064$
$-\frac{4}{3}\mathcal{A}_{\text{FB,Pol}}^{0,\tau} = -\mathcal{P}_e$	$0.1483 \pm 0.0051$	$\frac{4}{3}\mathcal{A}_{\text{FB,LR}}^{0,\mu} = -\mathcal{P}_\mu$	$0.137 \pm 0.016$
$\mathcal{A}_{\text{LR}}^0 = -\mathcal{P}_e$	$0.1511 \pm 0.0022$	$\frac{4}{3}\mathcal{A}_{\text{FB,LR}}^{0,\tau} = -\mathcal{P}_\tau$	$0.142 \pm 0.016$
$\{\frac{4}{3}\mathcal{A}_{\text{FB}}^{0,l}\}^{1/2} = -P_l$	$0.1506 \pm 0.0042$		

Table 4:  $\mathcal{P}_l$  determinations from different asymmetry measurements [10, 12, 13].

to the 0.15% level for the axial–vector neutral couplings, while only a few per cent precision has been achieved for the vector couplings [10, 12]:

$$\begin{aligned} a_\mu/a_e &= 1.0001 \pm 0.0014, & v_\mu/v_e &= 0.981 \pm 0.082, \\ a_\tau/a_e &= 1.0019 \pm 0.0015, & v_\tau/v_e &= 0.964 \pm 0.032. \end{aligned} \quad (5)$$

Assuming lepton universality, the measured leptonic asymmetries can be used to obtain the effective electroweak mixing angle in the charged–lepton sector [10, 12]:

$$\sin^2 \theta_{\text{eff}}^{\text{lept}} \equiv \frac{1}{4} \left( 1 - \frac{v_l}{a_l} \right) = 0.23119 \pm 0.00021 \quad (\chi^2/\text{d.o.f.} = 3.4/4). \quad (6)$$

## 4 Lorentz Structure

Let us consider the leptonic decay  $l^- \rightarrow \nu_l l'^- \bar{\nu}_{l'}$ . The most general, local, derivative–free, lepton–number conserving, four–lepton interaction Hamiltonian, consistent with locality and Lorentz invariance [14, 15],

$$\mathcal{H} = 4 \frac{G_{l'l}}{\sqrt{2}} \sum_{n,\epsilon,\omega} g_{\epsilon\omega}^n \left[ \bar{l}' \Gamma^n (\nu_{l'})_\sigma \right] \left[ \overline{(\nu_l)_\lambda} \Gamma_n l_\omega \right], \quad (7)$$

contains ten complex coupling constants or, since a common phase is arbitrary, nineteen independent real parameters which could be different for each leptonic decay. The subindices  $\epsilon, \omega, \sigma, \lambda$  label the chiralities (left–handed, right–handed) of the corresponding fermions, and  $n$  the type of interaction: scalar ( $I$ ), vector ( $\gamma^\mu$ ), tensor ( $\sigma^{\mu\nu}/\sqrt{2}$ ). For given  $n, \epsilon, \omega$ , the neutrino chiralities  $\sigma$  and  $\lambda$  are uniquely determined.

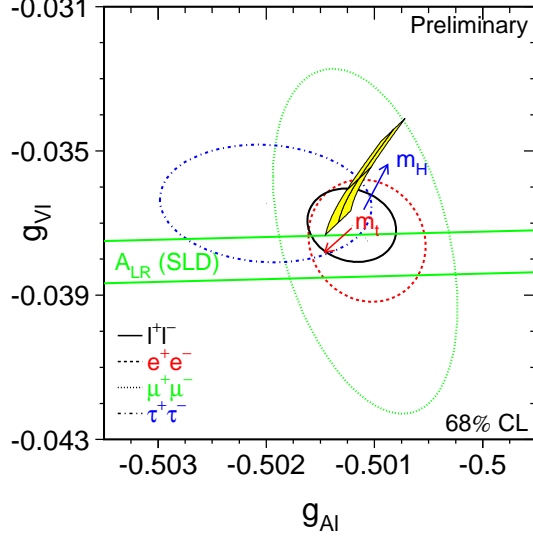


Figure 2: 68% probability contours in the  $a_l$ - $v_l$  plane from LEP measurements [10]. The solid contour assumes lepton universality. Also shown is the  $1\sigma$  band resulting from the  $\mathcal{A}_{LR}^0$  measurement at SLD. The shaded region corresponds to the SM prediction for  $m_t = 174.3 \pm 5.1$  GeV and  $m_H = 300_{-210}^{+700}$  GeV. The arrows show point in the direction of increasing  $m_t$  and  $m_H$  values.

The total decay width is proportional to the following combination of couplings, which is usually normalized to one [15]:

$$1 = \frac{1}{4} \left( |g_{RR}^S|^2 + |g_{RL}^S|^2 + |g_{LR}^S|^2 + |g_{LL}^S|^2 \right) + 3 \left( |g_{RL}^T|^2 + |g_{LR}^T|^2 \right) \quad (8)$$

$$+ \left( |g_{RR}^V|^2 + |g_{RL}^V|^2 + |g_{LR}^V|^2 + |g_{LL}^V|^2 \right) \quad (9)$$

$$\equiv Q_{LL} + Q_{LR} + Q_{RL} + Q_{RR}.$$

The universality tests mentioned before refer then to the global normalization  $G_{\nu l}$ , while the  $g_{\epsilon\omega}^n$  couplings parametrize the relative strength of different types of interaction. The sums  $Q_{\epsilon\omega}$  of all factors with the same subindices give the probability of having a decay from an initial charged lepton with chirality  $\omega$  to a final one with chirality  $\epsilon$ . In the SM,  $g_{LL}^V = 1$  and all other  $g_{\epsilon\omega}^n = 0$ .

The energy spectrum and angular distribution of the final charged lepton provides information on the couplings  $g_{\epsilon\omega}^n$ . For unpolarized leptons, the distribution is characterized by the so-called Michel [14] parameter  $\rho$  and the low-energy parameter  $\eta$ . Two more parameters,  $\xi$  and  $\delta$ , can be determined when the initial lepton polarization is known. In the SM,  $\rho = \delta = 3/4$ ,  $\eta = 0$  and  $\xi = 1$ .

For  $\mu$  decay, where precise measurements of the  $\mu$  and  $e$  polarizations have been performed, there exist [15] upper bounds on  $Q_{RR}$ ,  $Q_{LR}$  and  $Q_{RL}$ , and a lower bound on  $Q_{LL}$ . They imply corresponding upper limits on the 8 couplings  $|g_{RR}^n|$ ,  $|g_{LR}^n|$  and  $|g_{RL}^n|$ . The measurements of the  $\mu^-$  and the  $e^-$  do not allow to determine  $|g_{LL}^S|$  and  $|g_{LL}^V|$  separately; nevertheless, since the helicity of the  $\nu_\mu$  in pion decay is experimentally known to be  $-1$ , a lower limit on  $|g_{LL}^V|$  is obtained from the inverse muon decay  $\nu_\mu e^- \rightarrow \mu^- \nu_e$ . These limits show nicely that the bulk of the  $\mu$ -decay transition amplitude is indeed of the predicted V-A type:  $|g_{LL}^V| > 0.960$  (90% CL) [6].

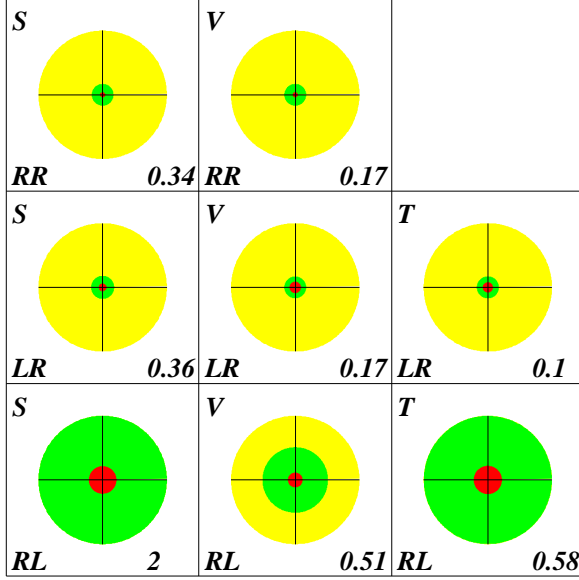


Figure 3: 90% CL experimental limits [16] for the normalized  $\tau$ -decay couplings  $g_{\epsilon\omega}^n \equiv g_{\epsilon\omega}^n/N^n$ , where  $N^n \equiv \max(|g_{\epsilon\omega}^n|) = 2, 1, 1/\sqrt{3}$  for  $n = S, V, T$ , assuming  $e/\mu$  universality. The circles of unit area indicate the range allowed by the normalization constraint. The present experimental bounds are shown as shaded circles. For comparison, the (stronger)  $\mu$ -decay limits are also shown (darker circles).

	$\mu \rightarrow e$	$\tau \rightarrow \mu$	$\tau \rightarrow e$	$\tau \rightarrow l$
$\rho$	$0.7518 \pm 0.0026$	$0.771 \pm 0.025$	$0.746 \pm 0.011$	$0.752 \pm 0.009$
$\eta$	$-0.007 \pm 0.013$	$0.173 \pm 0.122$	—	$0.035 \pm 0.031$
$\xi$	$1.0027 \pm 0.0085$	$1.053 \pm 0.053$	$0.995 \pm 0.042$	$0.978 \pm 0.031$
$\xi\delta$	$0.7506 \pm 0.0074$	$0.786 \pm 0.039$	$0.733 \pm 0.029$	$0.745 \pm 0.021$

Table 5: World average [16] Michel parameters. The last column ( $\tau \rightarrow l$ ) assumes identical couplings for  $l = e, \mu$ .

The experimental analysis of the  $\tau$ -decay parameters is necessarily different from the one applied to the muon, because of the much shorter  $\tau$  lifetime. The measurement of the  $\tau$  polarization and the parameters  $\xi$  and  $\delta$  is still possible due to the fact that the spins of the  $\tau^+\tau^-$  pair produced in  $e^+e^-$  annihilation are strongly correlated [11]. Another possibility is to use the beam polarization, as done by SLD. However, the polarization of the charged lepton emitted in the  $\tau$  decay has never been measured. The measurement of the inverse decay  $\nu_\tau l^- \rightarrow \tau^- \nu_l$  looks far out of reach.

The four LEP experiments, ARGUS, CLEO and SLD have performed accurate measurements of the  $\tau$ -decay Michel parameters. The present experimental status [16] is shown in Table 5. The determination of the  $\tau$  polarization parameters allows us to bound the total probability for the decay of a right-handed  $\tau$ ,

$$Q_{\tau_R} \equiv Q_{RR} + Q_{LR} = \frac{1}{2} \left[ 1 + \frac{\xi}{3} - \frac{16}{9}(\xi\delta) \right]. \quad (10)$$

At 90% CL, one finds (ignoring possible correlations among the measurements):

$$Q_{\tau_R}^{\tau \rightarrow \mu} < 0.047, \quad Q_{\tau_R}^{\tau \rightarrow e} < 0.054, \quad Q_{\tau_R}^{\tau \rightarrow l} < 0.032, \quad (11)$$

where the last value refers to the  $\tau$  decay into either  $l = e$  or  $\mu$ , assuming identical  $e/\mu$  couplings. These probabilities imply corresponding limits on all  $|g_{RR}^n|$  and  $|g_{LR}^n|$  couplings. Including also the information from  $\rho$  and  $\eta$ , one gets the (90% CL) bounds on the  $\tau$ -decay couplings shown in Fig. 3, where  $e/\mu$  universality has been assumed.

The probability  $Q_{\tau \rightarrow l_R} \equiv Q_{RL} + Q_{RR}$  for the  $\tau$  decay into a right-handed lepton could be investigated through the photon distribution [17] in the decays  $\tau^- \rightarrow \nu_\tau l^- \bar{\nu}_l \gamma$ . Owing to the large backgrounds, no useful limits can be extracted from the recent CLEO measurement of these radiative decays [18].

## 5 Lepton–Number Violation

In the minimal SM with massless neutrinos, there is a separately conserved additive lepton number for each generation. All present data are consistent with this conservation law. However, there are no strong theoretical reasons forbidding a mixing among the different leptons, in the same way as happens in the quark sector. Many models in fact predict lepton–flavour or even lepton–number violation at some level. Experimental searches for these processes can provide information on the scale at which the new physics begins to play a significant role.

Table 6 shows the most recent limits [19] on lepton–flavour and lepton–number violating decays of the  $\tau$ . Although still far away from the impressive bounds [6] obtained in  $\mu$  decay [ $Br(\mu^- \rightarrow e^- \gamma) < 4.9 \times 10^{-11}$ ,  $Br(\mu^- \rightarrow e^- e^+ e^-) < 1.0 \times 10^{-12}$ ,  $Br(\mu^- \rightarrow e^- \gamma \gamma) < 7.2 \times 10^{-11}$  (90% CL)], the  $\tau$ -decay limits start to put interesting constraints on possible new physics contributions.

$X^-$	U. L.	$X^-$	U. L.	$X^-$	U. L.	$X^-$	U. L.
$e^- \gamma$	2.7	$\mu^- \gamma$	1.1	$e^- \pi^+ \pi^-$	2.2	$\mu^- \pi^+ \pi^-$	8.2
$e^- e^+ e^-$	2.9	$\mu^- \mu^+ \mu^-$	1.9	$e^- \pi^+ K^-$	6.4	$\mu^- \pi^+ K^-$	7.5
$e^- e^+ \mu^-$	1.7	$\mu^- \mu^+ e^-$	1.8	$e^- K^+ \pi^-$	3.8	$\mu^- K^+ \pi^-$	7.4
$e^- \mu^+ e^-$	1.5	$\mu^- e^+ \mu^-$	1.5	$e^- K^+ K^-$	6.0	$\mu^- K^+ K^-$	15
$e^- \pi^0$	3.7	$\mu^- \pi^0$	4.0	$e^+ \pi^- \pi^-$	1.9	$\mu^+ \pi^- \pi^-$	3.4
$e^- \eta$	8.2	$\mu^- \eta$	9.6	$e^+ \pi^- K^-$	2.1	$\mu^+ \pi^- K^-$	7.0
$e^- \rho^0$	2.0	$\mu^- \rho^0$	6.3	$e^+ K^- K^-$	3.8	$\mu^+ K^- K^-$	6.0
$e^- K^{*0}$	5.1	$\mu^- K^{*0}$	7.5	$e^- \pi^0 \pi^0$	6.5	$\mu^- \pi^0 \pi^0$	14
$e^- \bar{K}^{*0}$	7.4	$\mu^- \bar{K}^{*0}$	7.5	$e^- \pi^0 \eta$	24	$\mu^- \pi^0 \eta$	22
$e^- \phi$	6.9	$\mu^- \phi$	7.0	$e^- \eta \eta$	35	$\mu^- \eta \eta$	60

Table 6: 90% CL upper limits (in units of  $10^{-6}$ ) on  $B(\tau^- \rightarrow X^-)$  [19]

## 6 The Tau Neutrino

$X$	ALEPH [7]	CLEO [20]	DELPHI [21]	OPAL [22]
$3\pi$	25.7	—	28	35.3
$3\pi\pi^0$	—	28	—	—
$5\pi$	23.1	30	—	43.2
Combined	18.2	28	28	27.6

Table 7: 95% CL upper limits on  $m_{\nu_\tau}$  (in MeV), from  $\tau^- \rightarrow \nu_\tau X^-$  events.

All observed  $\tau$  decays are supposed to be accompanied by neutrino emission, in order to fulfil energy–momentum conservation requirements. From a two–dimensional likelihood fit of the visible energy and the invariant–mass distribution of the final hadrons in  $\tau^- \rightarrow \nu_\tau X^-$  events, it is possible to set a bound on the  $\nu_\tau$  mass. The best limits, shown in Table 7, are obtained from modes with a significant probability of populating the hadronic–mass end–point region.

The present data are consistent with the  $\nu_\tau$  being a conventional sequential neutrino. Since taus are not produced by  $\nu_e$  or  $\nu_\mu$  beams, we know that  $\nu_\tau$  is different from the electronic and muonic neutrinos. LEP and SLC have confirmed [10] the existence of three (and only three) different light neutrinos, with standard couplings to the  $Z$ . However, no direct observation of  $\nu_\tau$ , that is, interactions resulting from  $\tau$  neutrinos, has been made so far. The DONUT experiment at Fermilab is expected to provide soon the first evidence of  $\tau$  neutrinos (produced through  $p + N \rightarrow D_s + \dots$ , followed by the decays  $D_s \rightarrow \tau^- \bar{\nu}_\tau$  and  $\tau^- \rightarrow \nu_\tau + \dots$ ), through the detection of  $\nu_\tau + N \rightarrow \tau + X$ . This is an important goal in view of the recent SuperKamiokande results suggesting  $\nu_\mu \rightarrow \nu_\tau$  oscillations with  $m_{\nu_\tau}^2 - m_{\nu_\mu}^2 \sim (0.05 \text{ eV})^2$ . This hypothesis could be corroborated making a long–baseline neutrino experiment with a  $\nu_\mu$  beam pointing into a far ( $\sim 700 \text{ Km}$ ) massive detector, able to detect the appearance of a  $\tau$ . The possibility to perform such an experiment is presently being investigated.

## 7 Hadronic Decays

The semileptonic decay modes  $\tau^- \rightarrow \nu_\tau H^-$  probe the matrix element of the left–handed charged current between the vacuum and the final hadronic state  $H^-$ . For the two–pion final state, the hadronic matrix element is parametrized in terms of the so–called pion form factor:

$$\langle \pi^- \pi^0 | \bar{d} \gamma^\mu u | 0 \rangle \equiv \sqrt{2} F_\pi(s) (p_{\pi^-} - p_{\pi^0})^\mu . \quad (12)$$

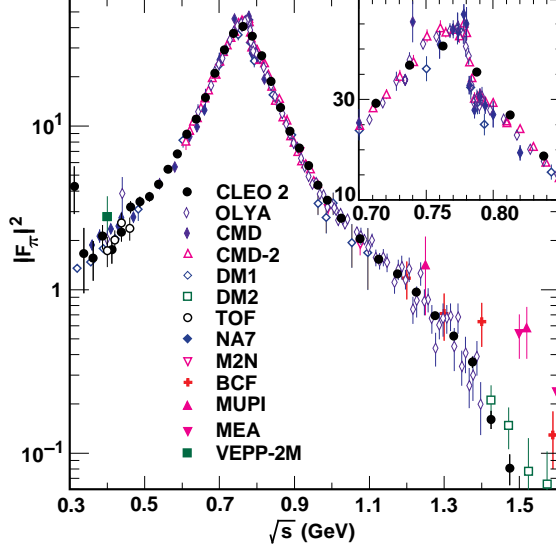


Figure 4: Pion form factor from  $\tau^- \rightarrow \nu_\tau \pi^- \pi^0$  data [23] (filled circles), compared with  $e^+e^- \rightarrow \pi^+\pi^-$  measurements.

Figure 4 shows the recent CLEO measurement [23] of  $|F_\pi(s)|^2$  from  $\tau \rightarrow \nu_\tau \pi^- \pi^0$  data (a similar analysis was done previously by ALEPH [24]). Also shown is the corresponding determination from  $e^+e^- \rightarrow \pi^+\pi^-$  data. The precision achieved with  $\tau$  decays is clearly better. There is a quite good agreement between both sets of data, although the  $\tau$  points tend to be slightly higher.

The dynamical structure of other hadronic final states has been also investigated. CLEO has measured recently [25] the four  $J^P = 1^+$  structure functions characterizing the decay  $\tau^- \rightarrow \nu_\tau \pi^- 2\pi^0$ , improving a previous OPAL analysis [26]. The interference between the two  $\pi^-\pi^0$  systems generates a parity-violating angular asymmetry [27], which allows to determine the sign of the  $\nu_\tau$  helicity to be  $-1$  (the modulus has been precisely measured through the study of correlated  $\tau^+\tau^-$  decays into different final states:  $|h_{\nu_\tau}| = 1.0000 \pm 0.0057$  [28]).

## 8 QCD Tests

The inclusive character of the total  $\tau$  hadronic width renders possible an accurate calculation of the ratio [29, 30]

$$R_\tau \equiv \frac{\Gamma[\tau^- \rightarrow \nu_\tau \text{ hadrons}(\gamma)]}{\Gamma[\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e(\gamma)]} = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}, \quad (13)$$

using analyticity constraints and the operator product expansion. One can separately compute the contributions associated with specific quark currents. Non-strange hadronic decays of the  $\tau$  are resolved experimentally into vector ( $R_{\tau,V}$ ) and axial-vector ( $R_{\tau,A}$ ) contributions according to whether the hadronic final state includes an

even or odd number of pions. Strange decays ( $R_{\tau,S}$ ) are of course identified by the presence of an odd number of kaons in the final state.

The theoretical prediction for  $R_{\tau,V+A}$  can be expressed as

$$R_{\tau,V+A} = N_C |V_{ud}|^2 S_{EW} (1 + \delta'_{EW} + \delta_P + \delta_{NP}), \quad (14)$$

with  $N_C = 3$  the number of quark colours. The factors  $S_{EW} = 1.0194$  and  $\delta'_{EW} = 0.0010$  contain the known electroweak corrections at the leading [3] and next-to-leading [31] logarithm approximation. The dominant correction ( $\sim 20\%$ ) is the purely perturbative QCD contribution [29, 30]

$$\delta_P = a_\tau + 5.2023 a_\tau^2 + 26.366 a_\tau^3 + \mathcal{O}(\alpha_s^4). \quad (15)$$

This expansion in powers of  $a_\tau \equiv \alpha_s(m_\tau^2)/\pi$  has rather large coefficients, which originate in the long running of the strong coupling along a contour integration in the complex plane; this running effect can be properly resummed to all orders in  $\alpha_s$  by fully keeping [30] the known four-loop-level calculation of the contour integral.

The non-perturbative contributions can be shown to be suppressed by six powers of the  $\tau$  mass [29], and are therefore very small. Their actual numerical size has been determined from the invariant-mass distribution of the final hadrons in  $\tau$  decay, through the study of weighted integrals [32],

$$R_{\tau,V+A}^{kl} \equiv \int_0^{m_\tau^2} ds \left(1 - \frac{s}{m_\tau^2}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{dR_{\tau,V+A}}{ds}, \quad (16)$$

which can be calculated theoretically in the same way as  $R_{\tau,V+A}$ . The predicted suppression [29] of the non-perturbative corrections has been confirmed by ALEPH [33], CLEO [34] and OPAL [35]. The most recent analyses [33, 35] give

$$\delta_{NP} = -0.003 \pm 0.003. \quad (17)$$

The QCD prediction for  $R_{\tau,V+A}$  is then completely dominated by the perturbative contribution  $\delta_P$ ; non-perturbative effects being smaller than the perturbative uncertainties from uncalculated higher-order corrections. The result turns out to be very sensitive to the value of  $\alpha_s(m_\tau^2)$ , allowing for an accurate determination of the fundamental QCD coupling. The experimental measurement [33, 35]  $R_{\tau,V+A} = 3.484 \pm 0.024$  implies  $\delta_P = 0.200 \pm 0.013$ , which corresponds (in the  $\overline{\text{MS}}$  scheme) to

$$\alpha_s(m_\tau^2) = 0.345 \pm 0.020. \quad (18)$$

The strong coupling measured at the  $\tau$  mass scale is significantly different from the values obtained at higher energies. From the hadronic decays of the  $Z$  boson, one gets  $\alpha_s(M_Z) = 0.119 \pm 0.003$ , which differs from the  $\tau$  decay measurement by eleven

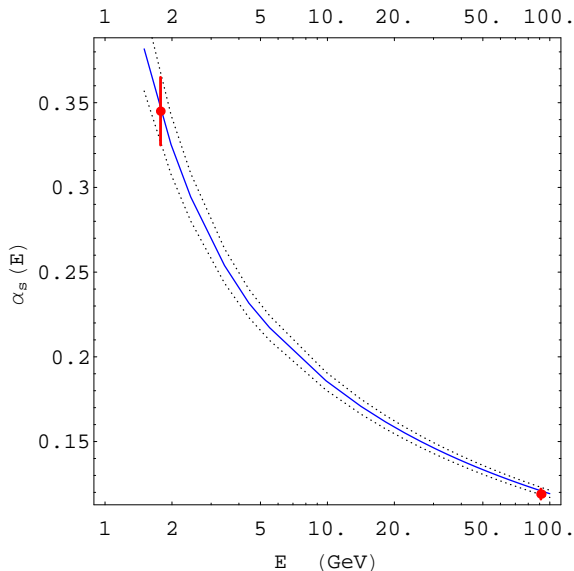


Figure 5: Measured values of  $\alpha_s$  in  $\tau$  and  $Z$  decays. The curves show the energy dependence predicted by QCD, using  $\alpha_s(m_\tau)$  as input.

standard deviations! After evolution up to the scale  $M_Z$  [36], the strong coupling constant in (18) decreases to

$$\alpha_s(M_Z^2) = 0.1208 \pm 0.0025, \quad (19)$$

in excellent agreement with the direct measurements at the  $Z$  peak and with a similar accuracy. The comparison of these two determinations of  $\alpha_s$  in two extreme energy regimes,  $m_\tau$  and  $M_Z$ , provides a beautiful test of the predicted running of the QCD coupling; i.e. a very significant experimental verification of *asymptotic freedom*.

From a careful analysis of the hadronic invariant-mass distribution, ALEPH [24, 33] and OPAL [35] have measured the spectral functions associated with the vector and axial-vector quark currents. Their difference is a pure non-perturbative quantity, which carries important information on the QCD dynamics; it allows to determine low-energy parameters, such as the pion decay constant, the electromagnetic pion mass difference  $m_{\pi^\pm} - m_{\pi^0}$ , or the axial pion form factor, in good agreement with their direct measurements.

The vector spectral function has been also used to measure the hadronic vacuum polarization effects associated with the photon and, therefore, estimate how the electromagnetic fine structure constant gets modified at LEP energies. The uncertainty of this parameter is one of the main limitations in the extraction of the Higgs mass from global electroweak fits to the LEP/SLD data. From the ALEPH  $\tau$  data [24], the Orsay group obtains [37]  $\alpha^{-1}(M_Z) = 128.933 \pm 0.021$ , which reduces the error of the fitted  $\log(M_H)$  value by 30%. The same  $\tau$  data allows to pin down the hadronic contribution to the anomalous magnetic moment of the muon  $a_\mu^\gamma$ . The recent analyses [23, 37] have improved the theoretical prediction of  $a_\mu^\gamma$ , setting a reference value to be

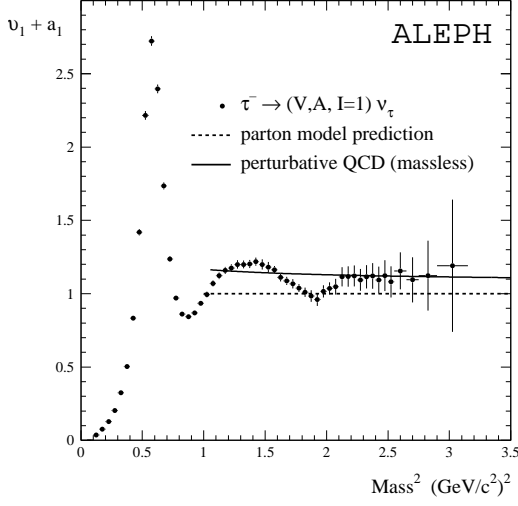


Figure 6:  $V + A$  spectral function [33].

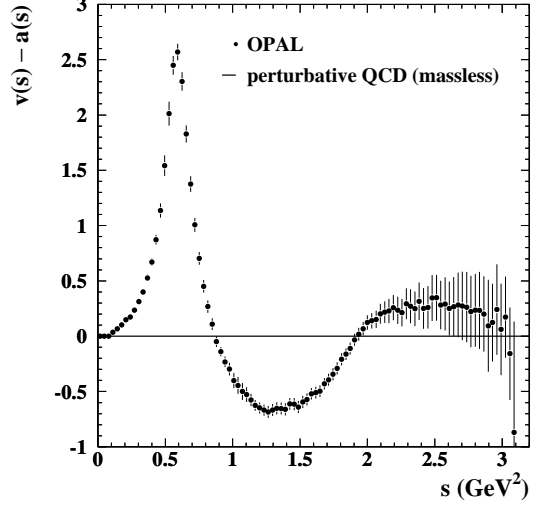


Figure 7:  $V - A$  spectral function [35].

compared with the forthcoming measurement of the BNL-E821 experiment, presently running at Brookhaven.

## 9 The Strange Quark Mass

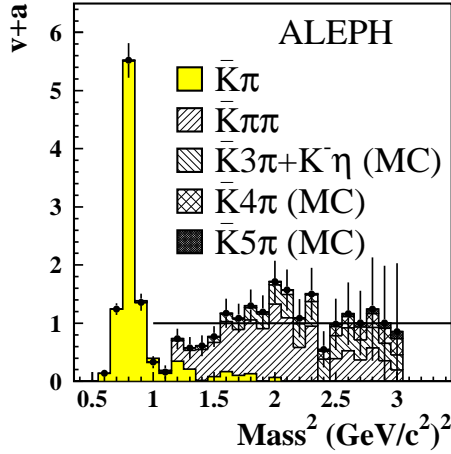
The LEP experiments and CLEO have performed an extensive investigation of kaon production in  $\tau$  decays. ALEPH has determined the inclusive invariant mass distribution of the final hadrons in the Cabibbo-suppressed decays [38]. The separate measurement of the  $|\Delta S| = 0$  and  $|\Delta S| = 1$  decay widths allows us to pin down the SU(3) breaking effect induced by the strange quark mass, through the differences

$$\delta R_{\tau}^{kl} \equiv \frac{R_{\tau,V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} \approx 24 \frac{m_s^2(m_{\tau})}{m_{\tau}^2} \Delta_{kl}(\alpha_s) - 48\pi^2 \frac{\delta O_4}{m_{\tau}^4} Q_{kl}(\alpha_s), \quad (20)$$

where  $\Delta_{kl}(\alpha_s)$  and  $Q_{kl}(\alpha_s)$  are perturbative QCD corrections, which are known to  $O(\alpha_s^3)$  and  $O(\alpha_s^2)$ , respectively [39]. The small non-perturbative contribution,  $\delta O_4 \equiv \langle 0 | m_s \bar{s}s - m_d \bar{d}d | 0 \rangle = -(1.5 \pm 0.4) \times 10^{-3} \text{ GeV}^4$ , has been estimated with Chiral Perturbation Theory techniques [39]. Table 8 shows the measured [38] differences  $\delta R_{\tau}^{kl}$  and the corresponding  $(\overline{\text{MS}})$  values [39] of  $m_s(m_{\tau})$ . The theoretical errors are dominated by the large perturbative uncertainties of  $\Delta_{kl}(\alpha_s)$ . Taking into account the information from the three moments, one finally gets [39]

$$m_s(m_{\tau}) = (119 \pm 12_{\text{exp}} \pm 18_{\text{th}} \pm 10_{V_{us}}) \text{ MeV}, \quad (21)$$

where the additional error reflects the present uncertainty from  $|V_{us}|$ . This corresponds to  $m_s(1 \text{ GeV}^2) = (164 \pm 33) \text{ MeV}$ .



$(k, l)$	$\delta R_\tau^{kl}$	$m_s(m_\tau)$ (MeV)
(0, 0)	$0.394 \pm 0.137$	$143 \pm 31_{\text{exp}} \pm 18_{\text{th}}$
(1, 0)	$0.383 \pm 0.078$	$121 \pm 17_{\text{exp}} \pm 18_{\text{th}}$
(2, 0)	$0.373 \pm 0.054$	$106 \pm 12_{\text{exp}} \pm 21_{\text{th}}$

Table 8: Measured moments  $\delta R_\tau^{kl}$  [38] and corresponding  $m_s(m_\tau)$  values [39].

Figure 8:  $|\Delta S| = 1$  spectral function [38].

## 10 Summary

The flavour structure of the SM is one of the main pending questions in our understanding of weak interactions. Although we do not know the reason of the observed family replication, we have learned experimentally that the number of SM fermion generations is just three (and no more). Therefore, we must study as precisely as possible the few existing flavours to get some hints on the dynamics responsible for their observed structure.

The  $\tau$  turns out to be an ideal laboratory to test the SM. It is a lepton, which means clean physics, and moreover it is heavy enough to produce a large variety of decay modes. Naïvely, one would expect the  $\tau$  to be much more sensitive than the  $e$  or the  $\mu$  to new physics related to the flavour and mass-generation problems.

QCD studies can also benefit a lot from the existence of this heavy lepton, able to decay into hadrons. Owing to their semileptonic character, the hadronic  $\tau$  decays provide a powerful tool to investigate the low-energy effects of the strong interactions in rather simple conditions.

Our knowledge of the  $\tau$  properties has been considerably improved during the last few years. Lepton universality has been tested to rather good accuracy, both in the charged and neutral current sectors. The Lorentz structure of the leptonic  $\tau$  decays is certainly not determined, but begins to be experimentally explored. An upper limit of 3.2% (90% CL) has been already set on the probability of having a (wrong) decay from a right-handed  $\tau$ . The quality of the hadronic data has made possible to perform quantitative QCD tests and determine the strong coupling constant very

accurately. Moreover, the Cabibbo-suppressed decay width of the  $\tau$  provides a rather good measurement of the strange quark mass. Searches for non-standard phenomena have been pushed to the limits that the existing data samples allow to investigate.

At present, all experimental results on the  $\tau$  lepton are consistent with the SM. There is, however, large room for improvements. Future  $\tau$  experiments will probe the SM to a much deeper level of sensitivity and will explore the frontier of its possible extensions.

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## Discussion

**Michel Davier (LAL, Orsay):** The value you quoted for  $m_s(m_\tau)$  from the ALEPH analysis is not the ALEPH result. Our analysis is more conservative and quotes considerably larger errors.

**Pich:** ALEPH [38] quotes two different results:  $(149_{-30\text{exp}}^{+24\text{exp}+21\text{th}} \pm 6_{\text{fit}})$  MeV and  $(176_{-48\text{exp}}^{+37\text{exp}+24\text{th}} \pm 8_{\text{fit}} \pm 11_{J=0})$  MeV. The first number is obtained truncating the longitudinal perturbative series  $\Delta_{kl}^L(\alpha_s)$  at  $O(\alpha_s)$ . The known  $O(\alpha_s^2)$  and  $O(\alpha_s^3)$  contributions are positive; thus, you took a smaller value of  $\Delta_{kl}(\alpha_s)$  and, therefore, got a larger result for  $m_s$ . Your error is larger to account for the missing  $O(\alpha_s^2)$  and  $O(\alpha_s^3)$  corrections.

The second number has been obtained subtracting the  $J = 0$  contribution; unfortunately, only the pion and kaon contributions are known. Since the longitudinal spectral functions are positive definite, this procedure gives an upper bound on  $m_s$  [39]. ALEPH makes a tiny ad-hoc correction to account for the remaining unknown  $J = L$  contribution, and quotes the resulting number as a  $m_s(m_\tau)$  determination. Since you added a generous uncertainty, your number does not disagree with ours. However, it is actually an upper bound on  $m_s(m_\tau)$  and not a determination of this parameter.